International Science and Technology Journal
المجلة الدولية للعلوم والتقنية

Volume 32 العدد
April 2023 ابريل

المجلة الدولية للعلوم والتقنية
International Science and Technology Journal
ISTJ

Efficient Numerical Solution of Airy Equations through Adomian Decomposition Method with MATLAB

Kareem Mustafi Shuayb
Faculty of oil, gas and renewable energy engineering
University of Zawia
k.shuayb@zu.edu.ly

الملخص:

تعد طريقة تحليل ادوميان من النظريات العددية القوية لحل المعادلات التفاضلية العادية والجزئية ويمكن حساب الحل بسهولة كمتسلسلة لانهائية، في هذه الورقة استعرضنا طريقة تحليل ادميان لحل معادلة أيري التفاضلية إضافة الى استخدام برنامج الماتلاب لتوضيح النتائج التي أوضحت بان الطريقة تقنية فعالة ودقيقة مقارنة بالحلول المضبوطة.

الكلمات الدليلية: المعادلة التفاضلية الهوائية، MATLAB، طريقة تحلل Anomia، Adomian كثير الحدود

Abstract

The Adomian decomposition method (ADM) is a powerful numerical technique for solving nonlinear partial differential equations in an infinite series with easily computable components. This paper presents the application of ADM to the Airy equation with MATLAB, a powerful and versatile programming language that has been widely used in scientific computing. We present a step-by-step guide on how to solve the Airy equation with ADM and MATLAB results shows the method is a precise and efficient technique compared to existing exact solutions by matching in the graph as we are going reviews.

Keywords: Airy differential equation, MATLAB, Adomia decomposition method, Adomian polynomial

Introduction

George Adomian (1923-1996) developed a successful method to solve nonlinear functional equations in the 1980s. The Adomian decomposition technique is the name given to this procedure

(ADM). The method is based on decomposing a nonlinear functional equation solution into a sequence of functions. Each term in the series is taken from a polynomial formed from an analytic function's power series expansion. ADM provides the solution in an infinite series with easily computable components [1],[2],[7].

The Airy equation is a second-order linear differential equation describing the behavior of an oscillating beam in an elastic medium. It was developed by John W. Airy in 1841 as an approximate model for the transverse vibrations of stretched strings and bars. This equation has played an important role in many physical applications such as wave propagation in elastic media, water waves, and the vibration of optical fibers. It is also used extensively in mathematical physics and engineering applications. The Airy functions, denoted by Ai(x), Bi(x), are two linearly independent solutions of Airy equation [5],[6].

Consider the linear ordinary differential equations in the form

$$Lu + Ru = g(x) \quad (1)$$

Where u is an unknown function, the linear differential operator L can be viewed as the highest-order derivative in the equation, R is the remainder of the differential operator, and $g(x)$ is an inhomogeneous term.

Which is

$$L = \frac{d^n}{dx^n} \quad \Rightarrow \quad L^{-1}(\#) = \int_0^x \int_0^x \dots \int_0^x (\#)\ dx\, dx \dots dx \quad (2)$$

Applying L^{-1} to both sides of (1) gives

$$u(x) = \delta_0 + L^{-1}g(x) - L^{-1}Ru \quad (3)$$

Where

$$\delta_0 = u(0) + xu'(0) + \frac{x^2}{2!}u''(0) + \frac{x^3}{3!}u'''(0) + \dots + \frac{x^n}{n!}u^{(n)}(0),$$

$$L = \frac{d^{n+1}}{dx^{n+1}}$$

The Adomian decomposition method admits the decomposition of the unknown function u in the form of an infinite series of components [3],[4]

$$u(x)=\sum_{n=0}^{\infty}u_n, \qquad (5)$$

And

$$\sum_{n=0}^{\infty}u_n=\delta_0+L^{-1}g(x)-L^{-1}R\left(\sum_{n=0}^{\infty}u_n\right). \qquad (6)$$

Consequently

$$u_0=\delta_0+L^{-1}g(x)$$
$$u_{n+1}=-L^{-1}Ru_n, \quad n\geq 0. \qquad (7)$$

Now apply the ADM to the following airy equation:

$$w''(x)-xw(x)=0 \qquad (8)$$

assuming the initial condition: $w(0)=\alpha\,, w'(0)=\beta$

We can write it in the following form

$$Lw(x)=xw(x) \qquad (9)$$

$$where\ L=\frac{d^2}{dx^2}$$

Operating L^{-1} on Both sides of above eq. To get

$$L^{-1}(Lw)=L^{-1}(xw) \qquad (10)$$

$$\int_0^x\int_0^x\frac{d^2w}{dx^2}\,dx\ \,dx=L^{-1}(xw) \qquad (11)$$

$$\int_0^x w'(x)\Big|_0^x\,dx=\ L^{-1}(xw) \qquad (12)$$

$$\int_0^x(\,w'(x)-w'(0))\,dx=\ L^{-1}(xw)\,,$$

$$where\ w'(0)\ a\ constant$$

$$w(x)-x\beta-\alpha=L^{-1}(xw) \qquad (13)$$

$$w(x)=\alpha+x\beta+L^{-1}(xw) \qquad (14)$$

The unidentified function $w(x)$ can be written in the form of an infinite series by ADM

$$w(x) = \sum_{n=0}^{\infty} w_n(x) \qquad (15)$$

As we know that w_n $'s$ are the Adomian polynomials and from (15) this results in

$$w_1(x) + w_2(x) + \cdots = L^{-1}\left(x \sum_{n=0}^{\infty} w_n(x)\right) \qquad (16)$$

Where

$$w_0(x) = \alpha + \beta x \qquad (17)$$

$$w_{n+1}(x) = L^{-1}\big(x\, w_n(x)\big)\ , \quad n \geq 0 \qquad (18)$$

Now

$$n = 0 \ : \ w_1(x) = L^{-1}\big(x\, w_0(x)\big)$$

$$= L^{-1}(\ \alpha\, x + \beta x^2)$$

$$= \int_0^x \int_0^x (\alpha\, x + \beta x^2)\ dx\ \ dx$$

$$= \frac{\alpha\, x^3}{6} + \frac{\beta\, x^4}{12} \qquad (19)$$

$$n = 1\,,\ \ w_2(x) = L^{-1}\big(x\, w_1(x)\big)$$

$$= \frac{\alpha\, x^6}{180} + \frac{\beta\, x^7}{504} \qquad (20)$$

This in turn gives

$$w(x) = w_0(x) + w_1(x) + w_2(x) + \cdots$$

$$= \alpha + \beta\, x + \frac{\alpha\, x^3}{6} + \frac{\beta\, x^4}{12} + \frac{\alpha\, x^6}{180} + \frac{\beta\, x^7}{504} + \cdots$$

$$= \alpha\left(1 + \frac{x^3}{6} + \frac{x^6}{180} + \cdots\right) + \beta\left(x + \frac{x^4}{12} + \frac{x^7}{504} + \cdots\right) \qquad (21)$$

more specifically

$$w_1(x) = \alpha\left(1 + \frac{x^3}{6} + \frac{x^6}{180} + \cdots\right) \qquad (22)$$

$$w_2(x) = \beta\left(x + \frac{x^4}{12} + \frac{x^7}{504} + \cdots\right) \qquad (23)$$

Are the basis form of system to solve the Airy's differential equation.

Example 1

Consider the following equation:

$y''(x) - xy(x) = 0$, where $y(0) = y'(0) = 1$ initial conditions

We get by using ADM the solutions

$$y_1(x) = 1 + \frac{x^3}{6} + \frac{x^6}{180} + \cdots$$
$$y_2(x) = x + \frac{x^4}{12} + \frac{x^7}{504} + \cdots$$

Now by using MATLAB we get

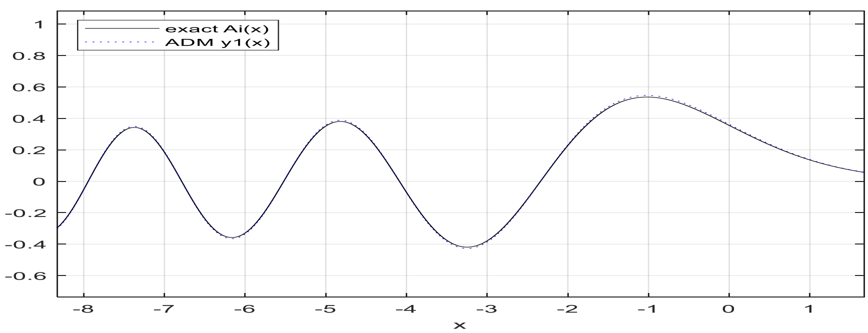

Figure 1: Comparison of the Ai(x) exact solution and the solution by ADM.

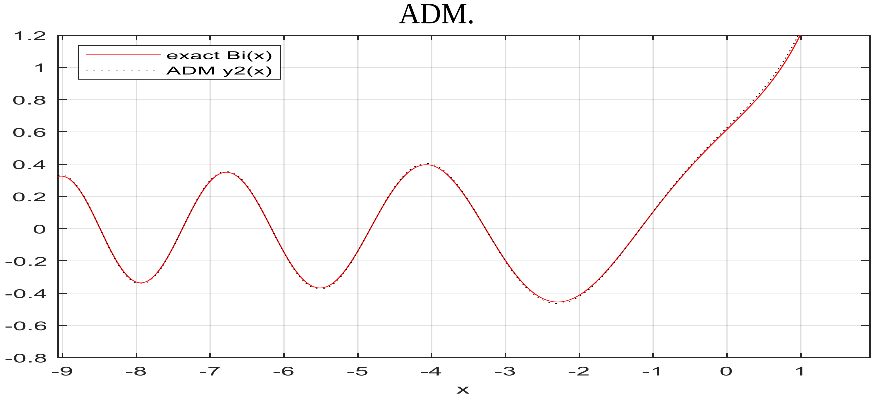

Figure 2: Comparison of the Bi(x) exact solution and the solution by ADM

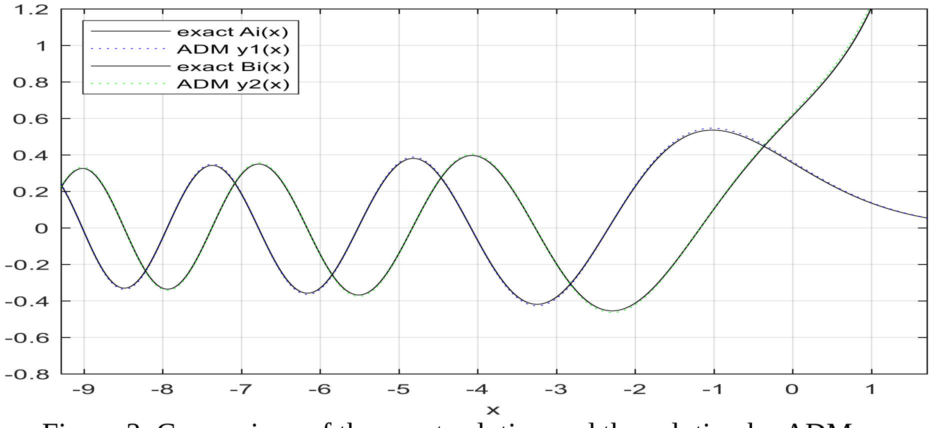

Figure 3: Comparison of the exact solution and the solution by ADM.

Results and Discussion

The Adomian Decomposition Method (ADM) is a powerful method that treats the approximate solution of a nonlinear equation as an infinite series that generally converges to the exact solution. In this article, we propose an efficient numerical solution of the Airy equations by Adomian decomposition in MATLAB. shows that the proposed method is very efficient and accurate in solving the Airy equations as we show in the graph matching between the solutions each type of Airy function given, (Figures 1 and 2) [9],[10], Figure 3: Comparison of the exact solution and the solution by ADM.

Conclusion

In this paper, we have presented the Adomian Decomposition Method (ADM) to solve the Airy equation with MATLAB. We have shown that ADM is an effective method to solve differential equations and can be used to solve a wide range of problems in physics and engineering. The results of our simulations show that ADM provides accurate solutions to the Airy equation and can be used to study the behavior of the Airy functions Ai(x) and Bi(x) for different values of x. Our work contributes to the growing body of literature on ADM and its applications in solving differential equations.

In conclusion, we have demonstrated that ADM is a powerful tool for solving differential equations and can be used to study a wide range of physical phenomena. Our work provides a foundation for future research in this area and highlights the potential of ADM as a tool for solving complex problems in physics and engineering.

References

[1]. Adomian, G. (1994). Solving Frontier Problems of Physics: The Decomposition Method. Boston: Kluwer Academic Publishers.

[2]. Wazwaz, A. M. (2006). Partial Differential Equations and Solitary Waves Theory. New York: Springer.

[3]. G. NHAWU_, P. MAFUTA, J. MUSHANYU, The Adomian Decomposition Method for numerical solution of first-order differential equation, J. Math. Comput. Sci. 6 (2016), No. 3, 307-314

[4]. Ahmed Abdelrazec,1 Dmitry Pelinovsky2, Convergence of the Adomian Decomposition Method for Initial-Value Problems *November 2009 in Wiley Online Library*.

[5]. M S Ramkarthik and Elizabeth Louis Pereira, Airy Functions Demystified-I, Vol.26, No.5, DOI, may 2021, https://doi.org/10.1007/s12045-021-1166-4

[6]. J.-C. Cortés ∗, L. Jodar, J.-V. Romero, M.-D. Rosello. A comparative study of the numerical approximation of the random Airy differential equation. Computers and Mathematics with Applications 62 (2011) 3411–3417

[7]. D. J. EVANS and K. R. RASLAN. THE ADOMIA DECOMPOSITION METHOD FOR SOLVING DELAY DIFFERENTIAL EQUATION. International Journal of Computer Mathematics · January 2004

[8]. F. Zullo: On the dynamics of the zeros of solutions of the Airy equation, Mathematics and Computers in Simulation, 176, 2020, pp.312-318

[9]. Dr. Abul Hasan Siddiqi's. The Airy's Equation. Lecture Notes on Differential Equation, Chapter 6. July 2013.

[10]. E Eroglu, N Ak, KO Koklu, ZO Ozdemir, N Celik, et al. Special Functions in Transferring of Energy; a special case: "Airy function". Energy Education Science and Technology, Part A-Energy Science and Research (ISI). 2012; 30: 719–726.